

Extensions of Green Periods for Freight Vehicles at Signalized Intersections: a Stochastic Analysis

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Abstract

The omnipresence of freight vehicles in road traffic has a big impact on traffic delays, pollution levels and road safety. For all these reasons it makes sense to avoid that freight vehicles have to stop at intersections. Vehicle-to-Infrastructure communication may act as a catalyst in the (near) future. In this paper, we study the impact of green period extensions for freight vehicles on the main road at signalized intersections. We quantify how much regular vehicles at the main road profit in terms of average waiting time and how much vehicles at the side road suffer. Moreover, our formulas allow for optimizing the length of the extension period.

Keywords: Signalized intersection, freight signal priority, waiting time.

1 Introduction

Freight transport via road networks has been continuously increasing in the last decades, absolutely as well as relatively. Trucks make for 75% of the total freight transport in Europe [1] and the share of freight vehicles in total traffic lies typically around 10% [2]. Another trend is Just-in-Time delivery, which pushes freight vehicles increasingly to urban roads and into cities. This obviously has a large impact on pollution levels, road safety, and traffic delays.

Especially decelerations and accelerations of freight vehicles have a large impact. Therefore, it makes sense to avoid these as much as possible. At signalized intersections, one way of doing this is by detecting arriving freight vehicles and trying to give them a green light upon their arrival at the intersection [2]. Detection of freight vehicles can be done through sensing traffic and distinguishing freight vehicles from regular vehicles (by means of height, weight, ...) or, in the near future, by means of V2I (Vehicle-to-Infrastructure) communication [2].

Giving green light to freight vehicles can be done by extending the green period when a freight vehicle is approaching [3]. Kari et al. [2] and Zhao and Ioannou [3] propose control policies that optimize a given objective function consisting of Measures of Effectiveness (MoEs) such as delays, fuel consumption and vehicle stopping probabilities. However, the necessary investments (detector infrastructure, V2I communication, ...) are only interesting if the fraction of freight vehicles is large enough. Therefore, we envisage this to be only useful for through traffic on a main road and not for freight vehicles coming from side roads. Furthermore, for efficiency, fairness, and/or safety reasons, it is not possible or preferred to extend this green period indefinitely. Therefore, freight vehicles may still get red lights occasionally.

In this paper, we propose and analyze a first simplified mathematical model for an intersection with possibility for green extension on the main road. Although some experimental and simulation work has been done, no analytical analysis has been performed to estimate the impact of such a policy. We assume that the green period for the main road is extended by a fixed time if a freight vehicle will arrive in that period. We make abstraction of all implementation issues. In this paper, we focus in particular on the impact of such a policy on the expected waiting time (lost time, wasted time, ...) of regular vehicles at the intersection. We show an excerpt of the analysis of regular vehicles on the main road. The analysis of the expected waiting time of vehicles on the side road is similar and therefore largely omitted. At the end, we show a small numerical example.

2 Model and Analysis

We analyze one direction on the main road. We assume freight vehicles arrive from that direction at the intersection according to a Poisson process with rate λ_f vehicles/second. Regular vehicles arrive at the intersection according to a Poisson process with rate λ_n vehicles/second. Green period can also be extended by freight vehicles arriving from the other direction on the main road. We assume that these arrive according to a Poisson process with rate $\hat{\lambda}_f$. In a regular red/green cycle¹, the duration of a red period is fixed and equal to t_r seconds, while the duration of a green period is also fixed and equal to t_g seconds. When freight vehicles would arrive at the intersection during the first t_{eg} seconds of the next red period, however, the green period is instead extended with t_{eg} seconds. We further assume that regular (freight respectively) vehicles drive at v_n (v_f respectively) m/s while accelerating after having stopped ($v_f < v_n$). A regular vehicle takes up l_n meters of the road when stopped at a traffic light, while a freight vehicle takes up l_f meters.

We make the following simplifying assumption: all vehicles that arrived during a red period can leave the intersection during the next regular green period. This assumption is accurate in light-traffic scenarios. We stress however that the problem we deal with is especially interesting in a light-traffic (to moderate-traffic) scenario anyway. We can then analyze the evolution of the expected waiting time in one cycle. We assume each cycle *ends* with a regular green period. So, if a green period is extended, we regard the extension of that green period to be the start of the next cycle. We thus distinguish two types of cycles, a regular cycle and an extended cycle that starts with an extended green period. The probability that a regular vehicle arrives in a cycle of the latter type equals

$$\frac{[1 - \exp(-(\lambda_f + \hat{\lambda}_f)t_{eg})](t_{eg} + t_r + t_g)}{[1 - \exp(-(\lambda_f + \hat{\lambda}_f)t_{eg})t_{eg} + t_r + t_g]},$$

where we used that a random cycle is an extended one if at least one freight vehicle arrives on the main road in the first t_{eg} seconds of the cycle. We show the analysis of the expected waiting time of a regular vehicle arriving in an extended cycle in detail. We assume the cycle starts at time 0 and we calculate the expected waiting time $E[W_n(t)]$ of a vehicle arriving at time t , $t \in [0, t_{eg} + t_r + t_g]$. Averaging over all t then leads to the expected waiting time of a vehicle arriving in an extended cycle.

If the vehicle arrives during the interval $[0, t_{eg}]$ (extended green period), it does not have to wait. So, $E[W_n(t)] = 0$, $t \in [0, t_{eg}]$.

¹We assume yellow periods are part of the red time.

The waiting time of a vehicle arriving in the red period consists of a remaining red time and the time to accelerate and pass the traffic light once it turns green. The latter component depends on the distance that the tagged vehicle is removed from the intersection and at what speed the vehicle can cover this distance. Denoting the number of regular vehicle arrivals (freight vehicle arrivals respectively) in an interval of length t by $U_n(t)$ ($U_f(t)$ respectively), we can write

$$\begin{aligned} E[W_n(t)] &= t_r + t_{eg} - t + \frac{l_f}{v_f} E[U_f(t - t_{eg})] + \frac{l_n}{v_n} E[U_n(t - t_{eg}) 1_{U_f(t - t_{eg})=0}] \\ &\quad + \frac{l_n}{v_f} E[U_n(t - t_{eg}) | 1_{U_f(t - t_{eg}) > 0}], \end{aligned}$$

$t \in [t_{eg}, t_{eg} + t_r]$. Here, 1_X is the indicator function of the event X . Since regular and freight vehicles arrive according to independent Poisson processes with rates λ_n and λ_f , we find

$$E[W_n(t)] = t_r + t_{eg} - t + \frac{[\lambda_n l_n + \lambda_f l_f](t - t_{eg})}{v_f} - \lambda_n l_n \left(\frac{1}{v_f} - \frac{1}{v_n} \right) (t - t_{eg}) \exp(-\lambda_f(t - t_{eg})),$$

$t \in [t_{eg}, t_{eg} + t_r]$.

Analyzing the waiting time of a regular vehicle arriving in the regular green interval is the most difficult part, since this waiting time depends on whether the queue caused by the red period has already dissolved at the moment of arrival. As long as it has not, newly arriving vehicles are delayed and add to the queue. However, once it has dissolved, the waiting time of newly arriving vehicles is zero. This intricate interplay makes the analysis difficult. Furthermore, whether or not a freight vehicle is part of the queue also plays a role. We propose a simple fluid-flow approximation. We start with the expected queue size built up during the red period. We assume that this queue is dissolved at speed v_n m/s as long as no freight vehicle is present, and at speed v_f m/s otherwise. Furthermore, newly arriving regular (freight respectively) vehicles add l_n (l_f respectively) meters to the queue as long as the queue has not completely dissolved.

We start with the expected queue size at the beginning of the green period and calculate the expected waiting time at time t , $t \in [t_{eg} + t_r, t_{eg} + t_r + t_g]$. We need to keep track of the presence of freight vehicles in the queue. We therefore split the calculation in three cases. In the first case, at least one freight vehicle arrives during the red period. In the second case, no freight vehicles arrive before time t . In the third case, no freight vehicles arrive in the red period, but at least one freight vehicle arrives before time t . This is the most intricate case, because the speed at which the queue dissolves switches from v_n to v_f m/s when the first freight vehicle arrives (if the queue has not dissolved by then). Denoting the time between the start of the red period and the first arrival of a freight vehicle as I_f , we can write for the first case $((\cdot)^+ \triangleq \max(\cdot, 0))$

$$\begin{aligned} E[W_n(t) 1_{I_f \leq t_r}] &\approx \frac{(E[l_n U_n(t_r) + l_f U_f(t_r) | U_f(t_r) > 0] - (v_f - \lambda_f l_f - \lambda_n l_n)(t - t_{eg} - t_r))^+}{v_f} \Pr[I_f \leq t_r], \end{aligned}$$

since at least one freight vehicle arrived during the red period in this case, the queue dissolves at a net constant speed of $v_f - \lambda_f l_f - \lambda_n l_n$, the arriving vehicle can drive at speed v_f when arriving before dissolution of the queue, and the waiting time of the arriving vehicle is zero if the queue is already dissolved at the moment of arrival. We can further write

$$E[W_n(t) 1_{I_f \leq t_r}] \approx \frac{[(\lambda_n l_n(t - t_{eg}) - (v_f - \lambda_f l_f)(t - t_{eg} - t_r))(1 - \exp(-\lambda_f t_r)) + \lambda_f l_f t_r]^+}{v_f}.$$

The second case can be approximated in much the same way:

$$E[W_n(t)1_{t-t_{eg}<I_f}] \approx \frac{[\lambda_n l_n(t-t_{eg}) - v_n(t-t_{eg}-t_r)]^+}{v_n} \exp(-\lambda_f(t-t_{eg})). \quad (1)$$

The calculation of the last case is more involved. We first split the green period in three parts for this calculation. In the first interval $[t_{eg}+t_r, t_n]$, the queue is not dissolved, not even when no freight vehicles arrived before that time. The end point t_n of this interval can be calculated as

$$t_n = t_{eg} + \frac{v_n}{v_n - \lambda_n l_n} t_r.$$

As end result, we have

$$\begin{aligned} E[W_n(t)1_{t_r < I_f \leq t-t_{eg}}] \\ \approx \frac{(\lambda_n l_n + \lambda_f l_f - v_f)(t-t_{eg}) + v_n t_r + l_f}{v_f} (\exp(-\lambda_f t_r) - \exp(-\lambda_f(t-t_{eg}))) \\ + \frac{v_n - v_f + \lambda_f l_f}{\lambda_f v_f} [(1 + \lambda_f(t-t_{eg})) \exp(-\lambda_f(t-t_{eg})) - (1 + \lambda_f t_r) \exp(-\lambda_f t_r)], \end{aligned}$$

for $t \in [t_{eg}+t_r, t_n]$. In the second interval $[t_n, t_f]$, the waiting time is zero if the queue dissolves at speed v_n but non-zero if it dissolves at speed v_f (almost) directly from the beginning. The point t_f can be calculated as

$$t_f = t_{eg} + \frac{l_f + (v_f - \lambda_f l_f)t_r}{v_f - \lambda_n l_n - \lambda_f l_f}.$$

We have for $t \in [t_n, t_f]$

$$\begin{aligned} E[W_n(t)1_{t_r < I_f \leq t-t_{eg}}] \\ \approx \frac{(\lambda_n l_n + \lambda_f l_f - v_f)(t-t_{eg}) + v_n t_r + l_f}{v_f} (\exp(-\lambda_f t_r) - \exp(-\lambda_f T^*(t))) \\ + \frac{v_n - v_f + \lambda_f l_f}{\lambda_f v_f} [(1 + \lambda_f T^*(t)) \exp(-\lambda_f T^*(t)) - (1 + \lambda_f t_r) \exp(-\lambda_f t_r)], \end{aligned}$$

with

$$T^*(t) = \frac{v_n t_r + l_f - (v_f - \lambda_n l_n - \lambda_f l_f)(t-t_{eg})}{v_n - v_f + \lambda_f l_f}.$$

Finally, in the last interval $[t_f, t_{eg}+t_r+t_g]$, the queue has dissolved even if a freight vehicle arrived from the very start of the green period. As a result

$$E[W_n(t)1_{t_r < I_f \leq t-t_{eg}}] \approx 0,$$

for $t \in [t_f, t_{eg}+t_r+t_g]$.

Calculation of average waiting times conditional on arriving in an extended cycle is straightforward by averaging over all t .

The expected waiting time of a vehicle arriving in a regular cycle can be analyzed in much the same way. The difference is twofold: (i) this type of cycle exists of a red period and a regular green

period only, and (ii) no freight vehicle arrives during the first t_{eg} seconds of the cycle (otherwise the cycle would be an extended cycle). We find similar expressions as for the extended-cycle case (details are omitted). The expected waiting time of regular vehicles arriving from the side road is also similar. We merely have to take into account that extended green time in the main direction leads to extended red time in the side direction. We thus distinguish two types of cycles (regular and extended) for the side road as well.

3 Numerical Example

In figure 1, we show the average waiting time of regular vehicles of one direction on the main road and of one on the side road as function of the length of the extension of the green period. On average 0.25 regular vehicles per second arrive in the analyzed direction on the main road, 0.01 freight vehicles per second arrive from each direction on the main road, 10% of these numbers arrive on the side roads. Regular green and red periods on the main road equal 22 seconds, while those for the side road equal 18 and 26 seconds respectively. Regular vehicles are 5 meters long while freight vehicles have a length of 10 meters. It is shown that we can nicely calculate the impact of green period extensions on average waiting times of regular vehicles.

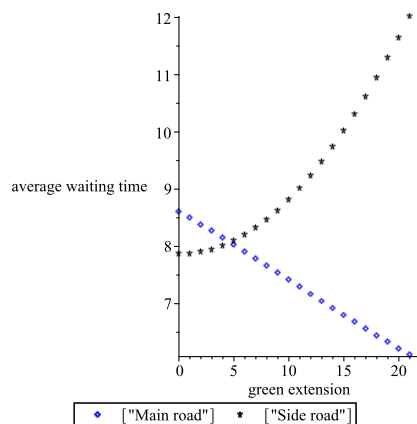


Figure 1: Average Waiting Time versus the Extension of the Green Period

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